

SYMPLECTIC NEWTON–COTES PRODUCT INTEGRATION METHODS FOR FRACTIONAL HAMILTONIAN SYSTEMS

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This work presents a novel class of composite product-integration closed Newton–Cotes (CNC) integrators specifically designed for Hamiltonian systems governed by Caputo fractional derivatives of order $1 < \alpha < 2$. The proposed methodology establishes explicit full-history convolution weights while constructing a discrete variational principle within an extended phase space, ultimately yielding a symplectic mapping. Through rigorous analysis, we demonstrate global error bounds of $O(h^{2-\alpha})$ for CNC-2 and $O(h^{4-\alpha})$ for CNC-4 under conventional regularity assumptions. The linear stability characteristics are analyzed via the generating function of convolution weights, while long-term energy drift behavior is quantified. Additionally, efficient implementation strategies are developed that achieve $O(1)$ per-step computational complexity through systematic history truncation and sum-of-exponentials compression techniques. Comprehensive numerical experiments validate the theoretical predictions and demonstrate superior performance relative to existing fractional integrators.

1. Introduction

Fractional differential equations have emerged as powerful mathematical tools in contemporary physics, engineering, and computational sciences, primarily due to their inherent capability to model systems exhibiting memory effects and non-local phenomena [14, 5].

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