

QUADRICS OF FINITE CHEN TYPE REGARDING THE SECOND FUNDAMENTAL FORM

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This paper focuses on the study of quadric surfaces in 3-dimensional Euclidean space that are of finite II -type, a concept introduced by B.-Y. Chen regarding the 2nd fundamental form II . It was shown that the only ones of the class of quadrics in 3-space according to the definition of finite Chen type are the spheres.

1. Introduction

Finite type immersions in Euclidean space have played a significant role in classifying and characterizing various well-known Riemannian submanifolds. In [8], B.-Y. Chen posed the problem of classifying finite type submanifolds in 3-dimensional Euclidean space $\mathbb{R}^3$. Since then, the theory of submanifolds of finite type has been extensively studied by many geometers.

Denote by $\mathbb{R}^3$ the 3-dimensional Euclidean space occupied with the metric

$$\langle d\mathbf{y}, d\mathbf{y} \rangle = dy_1^2 + dy_2^2 + dy_3^2,$$

where $\mathbf{y} = (y_1, y_2, y_3)$ is an arbitrary vector in $\mathbb{R}^3$.

Let $\mathbf{y} := \mathbf{y}(u^1, u^2)$ be a regular parametric representation of a surface Π in $\mathbb{R}^3$, and

$$\mathbf{G} = \frac{\frac{\partial \mathbf{y}}{\partial u^1} \times \frac{\partial \mathbf{y}}{\partial u^2}}{\left\| \frac{\partial \mathbf{y}}{\partial u^1} \times \frac{\partial \mathbf{y}}{\partial u^2} \right\|},$$

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