

## GENERALIZED FUZZY IDEALS IN ORDERED TERNARY SEMIGROUPS

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This work explores and defines generalized fuzzy left ideals, fuzzy right (lateral) ideals, fuzzy fuzzy bi-ideals, and quasi-ideals within the framework of ordered ternary semigroups, using a set-theoretic approach. Generalization of these fuzzy ideals is developed by employing Tom Head's metatheorem as a foundational tool. Head's metatheorem, which provides a unifying framework for transferring results from crisp algebraic systems to their fuzzy counterparts, serves as the foundational tool for our analysis. It is demonstrated that the classes of generalized fuzzy left (right, lateral) ideals, generalized fuzzy bi-ideals (quasi-ideals) and various types of fuzzy ideals are projection closed within an ordered ternary semigroup. Characterizations of generalized fuzzy bi-ideals and fuzzy quasi-ideals in terms of generalized fuzzy left (right, lateral) ideals are also provided. Tom Head's metatheorem is applied to derive proofs for numerous propositions related to these various types of generalized fuzzy ideals, effectively simplifying the proof process by avoiding intricate calculations. Several important properties of fuzzy substructures in ordered ternary semigroups have been established. The intersection of a fuzzy subsemigroup and a generalized fuzzy bi-ideal is shown to be a generalized fuzzy bi-ideal. Similarly, the product of three generalized fuzzy bi-ideals, as well as the product of three generalized fuzzy quasi-ideals, results in a generalized fuzzy bi-ideal. Furthermore, every generalized fuzzy quasi-ideal is a generalized fuzzy bi-ideal, and every generalized fuzzy bi-ideal of a regular ordered ternary semigroup is a generalized fuzzy quasi-ideal. Additionally, the intersection of a generalized fuzzy left ideal, generalized fuzzy lateral ideal, and generalized fuzzy right ideal is a generalized fuzzy quasi-ideal. By situating fuzzy ideal theory in the richer setting of

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