

A PRELIMINARY NOTE ON A MIXED FINITE DIFFERENCE APPROACH FOR A SINGULARLY PERTURBED GAIT MODEL

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In this research, we introduce a simulation based numerical technique for addressing a class of linear second-order ordinary differential equations derived from a simplified biologically inspired model of human gait. Although the governing equation is posed as a time-dependent ODE, its inner rescaled formulation exhibits singular perturbation structure characteristic of boundary layer type behavior. The model incorporates significant physical aspects like gravity, damping, and leg stiffness, while also illustrating the vertical motion of the body's center of mass during ambulation or running. Standard numerical schemes may struggle to accurately resolve steep solution gradients arising for small perturbation parameters. To address this, we employ an asymptotic inner-outer decomposition combined with a time rescaling transformation to capture boundary layer behavior effectively. The Thomas approach is used to quickly solve the resulting tridiagonal problems within the mixed finite difference framework. The numerical experiments are also presented in order to validate the theoretical findings.

1. Introduction

The vertical movement of the body's center of mass (COM) is the most significant part of this modeling. This is critical for both the energy efficiency and mechanical stability of movement. Basic mechanical models, such as spring-mass systems, inverted pendulum models, and their modifications, have effectively simulated critical

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