

ON A GENERALIZATION OF D3-MODULES

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R is supposed to be an associative ring with identity and M be a unital left R -module. In this paper, we present the concept of a $D31$ -module that generalizes the concept of a $D3$ -module introduced in [14] and studied in [25]. A module M is termed as $D31$ -module if, whenever W and X are summands of M with $W + X = M$ and W is cosingular, we have $W \cap X$ is a direct summand of M . We explore fundamental properties of these modules and establish that the class of rings R such that every $D31$ -module is also a $D3$ -module coincides precisely with the class of COSP-rings. Additionally, we examine the connections between $D31$ -modules and other related module classes.

1. Introduction

Throughout the manuscript, all rings R are considered to be associative with unity. We introduce some notations that will be used throughout. Given an R -module M , $N \leq M$ indicates that N is a submodule of M , and $N \leq^\oplus M$ denotes that N is a direct summand of M . Unless stated otherwise, all modules considered are left modules. Additionally, we use $N \subseteq M$ for the inclusion set. We abbreviate finitely generated by FG .

A submodule K of M is small in M (written $K \ll M$) if $K + H \neq M$ for every proper submodule H of M . A module M is hollow if every proper submodule of M is small in M [14, Definition 4.1]. In 2002,

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