

A GRID-BASED METHOD FOR SOLVING THE INITIAL-BOUNDARY VALUE PROBLEM FOR A MULTIDIMENSIONAL THIRD-ORDER HYPERBOLIC PARTIAL DIFFERENTIAL EQUATION

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The initial boundary value problem for a multidimensional partial differential equation of the third order of hyperbolic type, which serves as a mathematical model of the movement of moisture and salts in soils, is studied. For an approximate solution of the problem, the original equation is reduced to an integro-differential equation with a small parameter. It is shown that when the small parameter tends to zero, the solution of the resulting modified problem converges to the solution of the original problem. A. A. Samarsky's locally one-dimensional difference scheme is constructed. An a priori estimate is obtained using the method of energy inequalities, from which the uniqueness and stability of the solution of the scheme follow, the convergence of the solution of the locally one-dimensional difference scheme to the solution of the modified differential problem is proved.

1. Introduction

At present, local and non-local boundary value problems for third-order hyperbolic equations are being actively studied and are of great practical and theoretical interest due to the fact that applied problems in physics, mechanics, and biology are reduced to such equations. It is known that issues of liquid filtration in porous media [1], [2], heat transfer in a heterogeneous medium [3], [4], moisture transfer in soils and grounds [5], [6], problems of calculating heat

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