

ISO $C2$ AND ISO $C3$ -MODULES

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In this paper, we introduce and study the concept of iso $C2$ -modules and iso $C3$ -modules as generalizations of the direct injective modules. We prove that M is an iso $C3$ -module if for any two isosimple submodules A, B of M with $A \cong B$, $B \subseteq^{\oplus} M$ and $A \cap B = 0$ implies that $A \subseteq^{\oplus} M$. Also, if M is an iso $C3$ -module then $Imf \subseteq^{\oplus} B$, whenever $M = A \oplus B$ with A, B isosimple and $f : A \rightarrow B$ be an R -homomorphism. Furthermore, we provide a number of properties and characterizations of these new classes of modules.

1. Introduction

Let R be an associative ring with identity and M be an unital right R -module. Recall [4], M is a $C1$ -module if every submodule of M is essential in a direct summand of M and M is a $C2$ -module (also called direct injective module) if any submodule of M which is isomorphic to a direct summand of M is itself a direct summand of M . Also M is a $C3$ -module if whenever A and B are direct summand of M with $A \cap B = 0$, $A \oplus B$ is also a direct summand of M . Many authors have generalized the concept of direct injective modules and transferred several results of this class of modules (see [2, 4, 12, 14, 18, 19]). Recall [11], M is isosimple if every nonzero submodule of M is isomorphic to M . An isosimple submodule N of M means N is isosimple as R -module (see [7, 8]). Recently, Camilo et al. [6] introduced the notion of simple direct injective modules as a

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